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Final state interactions and $\Delta S = -1$, $\Delta C = \pm 1$ B-decays

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Abstract: The final state interactions (FSI) in $\Delta S = -1$, $\Delta C = \pm 1$ decays of B-meson are discussed. The rescattering corrections are found to be of order of $15-20\%$. The strong interaction phase shifts are estimated and their effects on CP- asymmetry are discussed.

Keywords: B-Physics, Heavy Quarks Physics.

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Contents

1. Introduction

Direct CP-violation in the decays $B \to f$ and $\bar{B} \to \bar{f}$ depends on the strong final state interactions. In fact the CP limit of no strong phase shifts. The purpose of this paper is to study the $\Delta C = \pm 1$, $\Delta S = -1$ B the final state interactions [1]–[5]. Such decays are described by the effective lagrangians

$$\begin{aligned}\mathcal{L}_{\mathrm{eff}} &= \frac{G_F}{\sqrt{2}} V_{cb} V_{us}^* [\bar{s}\gamma^\mu (1-\gamma_5) u][\bar{c}\gamma_\mu (1-\gamma_5) b] \\ \mathcal{L}_{\mathrm{eff}} &= \frac{G_F}{\sqrt{2}} V_{ub} V_{cs}^* [\bar{s}\gamma^\mu (1-\gamma_5) c][\bar{u}\gamma_\mu (1-\gamma_5) b] \,. \end{aligned} \tag{1.1}$$

Both these lagrangians have $\Delta I = 1/2$. The weak phase in the Wolfenstein parameterization [6] of CKM matrix [7] is given by

$$\frac{V_{ub} V_{cs}^*}{V_{cb} V_{us}^*} = \sqrt{\rho^2 + \eta^2} e^{i\gamma} \,, \qquad \sqrt{\rho^2+\eta^2} = 0.36 \pm 0.09 \,. \tag{1.2}$$

It is quite difficult to reliably estimate the final state interactions in weak decays. The problem is somewhat simplified by using isospin and SU(3) symmetry in discussing the strong interaction effects. We will fully make use of these symmetries. Moreover we note in Regge phenomenology, the strong interactions scattering amplitudes can be written in terms of Pomeron exchange and exchange of $\rho - A_2$ and $\omega - f$ trajectories in t -channel. The problem is further simplified if there is an exchange degeneracy. In fact this is the case here. In $s(u)$ channel only the states with quark structure $s\bar{c}$ ($c\bar{s}$) can be exchanged (s, u, t, are Mandelstam variables). An important consequence of this is that since $\bar{K}D$ has quantum numbers $C = 1$, $S = -1$, only a state with structure $c\bar{s}$ ($C = 1$, $S = 1$, $Q = +1$) can be exchanged in u-channel where as no exchange is allowed in the s-channel (exotic). On the other hand $\bar{K}\bar{D}$ ($C = -1$, $S = -1$) state is non-exotic and we can have an exchange of a state with structure $s\bar{c}$ ($C = -1$, $S = -1$, $Q = -1$) in s-channel and a state with structure $d\bar{c}$ or $u\bar{c}$ ($C = -1$, $S = 0$) in u -channel for a quasi-elastic channel such as $\bar{K}^0\bar{D}^0 \to \pi^+ D_s^-$ or $K^-\bar{D}^0(\bar{K}^0 D^-) \to \pi^0 D_s^-$. In Regge Phenomenology, exotic u-channel implies exchange degeneracy i.e. in t-channel $\rho - A_2$ and $\omega - f$ trajectories are exchange degenerate. Taking into account $\rho - \omega$ degeneracy, all the elastic or quasielastic scattering amplitudes can be expressed in terms of two amplitudes which we denote by F_P, F_ρ and $\bar{F}_\rho = e^{-i\pi\alpha(t)}F_\rho$, where F_P is given by pomeron exchange, F_ρ by particle exchange trajectory for which $\alpha_\rho(t) = \alpha_{A_2}(t) = \alpha_\omega(t) = \alpha_f(t) = \alpha(t)$. We have argued above, that elastic and quasi-elastic scattering amplitudes can be calculated fairly accurately. This combined with the following physical picture [8, 9] gives us a fairly reliable method to estimate the effect of rescattering on weak decays. In the weak decays of B-measons, the b quark is converted into $b \to c + q + \bar{q}$, $b \to u + q + \bar{q}$; since for the [8, 9] tree graph the configuration is such that q and $\bar{q}$ essentially go together into the color singlet state with the third quark recoiling, there is a significant probability that the system will hadronize as a two body final state. This physical picture has been put on a strong theoretical basis in [12, 10]. In this picture the strong phase shifts are expected to be small at least for tree amplitude.

Now the discontinuity or imaginary part of decay amplitude is given by [11, 13]

$$\mathrm{Im}\, A_f = \sum_n M_{fn}^* A_n \,, \tag{1.3}$$

where M_{fn} is the scattering amplitude for $f \to n$. According to above picture the important contribution to the decay amplitude $A_f \equiv A(B \to f)$ in eq. (1.3) is from those two body decays of B which proceed through tree graphs. Thus it is reasonable to assume that the decays proceeding through the following chains

$$\begin{aligned} \bar{B}^0 &\to K^- D^+ \to \bar{K}^0 D^0 \\ \bar{B}_s^0 &\to K^- D_s^+ \to \pi^- D^+ \end{aligned} \tag{1.4}$$

and

$$\begin{aligned} \bar{B}^0 &\to \pi^+ D_s^- \to \bar{K}^0 \bar{D}^0 \\ B^- &\to \pi^0 D_s^- \to \bar{K}^0 D^- \\ &\to \eta D_s^- \to K^- \bar{D}^0 \\ \bar{B}_s^0 &\to K^+ D_s^- \to \pi^+ D^- \end{aligned} \tag{1.5}$$

may make a significant contribution to the decay amplitudes. Note that the intermediate channels are those channels for which the decay amplitude is given by tree amplitude for which the factorization anstaz is on a strong footing [12, 10].

In view of above arguments, the dominant contribution in eq. (1.3) is from a state $n = \acute{f}$ where the decay amplitude $A_{\acute{f}}$ is given by the tree graph. For the quasi elastic channels listed in eqs. (1.4) and (1.5), the scattering amplitude $M_{f\acute{f}}$ is given by the ρ-exchange amplitude F_ρ or $\bar{F}_\rho$. The purpose of this paper is to calculate the rescattering

Mode	Amplitude	$\mathrm{A}_{\text{topology}}$
$B^- \to K^- D^0$	$A_{-0} = a_{-0} e^{i\delta_{-0}} = 2A_1$	$Te^{i\delta_T} + Ce^{i\delta_C}$
$\bar{B}^0 \to K^- D^+$	$A_{-+} = a_{-+} e^{i\delta_{-+}} = A_1 + A_0$	$Te^{i\delta_T}$
$\bar{B}^0 \to \bar{K}^0 D^0$	$A_{00} = a_{00} e^{i\delta_{00}} = A_1 - A_0$	$Ce^{i\delta_C}$
$\bar{B}^0_s \to K^- D^+_s$	$B_{-s+} = b_s e^{i\delta_s}$	$Te^{i\delta_T} + Ee^{i\delta_E}$
$\bar{B}^0_s \to \pi^- D^+$	$B_{-+} = b_{1/2} e^{i\delta_{1/2}} = B_{1/2}$	$Ee^{i\delta_E}$
$\bar{B}^0_s \to \pi^0 D^0$	$B_{00} = (1/\sqrt{2}) b_{1/2} e^{i\delta_{1/2}} = (1/\sqrt{2}) B_{1/2}$	$(1/\sqrt{2}) Ee^{i\delta_E}$

Table 1: Amplitudes for $\Delta C = \pm 1$ and $\Delta S = -1$ decay modes of $\bar{B}$. $\Delta C = +1$, $\Delta S = -1$ decays.

Mode	Amplitude	$\mathrm{A}_{\text{topology}}$
$\bar{B}^0 \to \bar{K}^0 D^0$	$\bar{A}_{00} = \bar{a}_{00} e^{i\left(\bar{\delta}_{00}+\gamma\right)} = 2\bar{A}_1$	$\bar{C} e^{i\left(\bar{\delta}_C+\gamma\right)}$
$B^- \to K^- \bar{D}^0$	$\bar{A}_{-0} = \bar{a}_{-0} e^{i\left(\bar{\delta}_{-0}+\gamma\right)} = \bar{A}_1 + \bar{A}_0$	$\left(\bar{C} e^{i\bar{\delta}_C} + \bar{A} e^{i\bar{\delta}_A}\right) e^{i\gamma}$
$\bar{B}^0 \to \bar{K}^0 D^-$	$\bar{A}_{0-} = \bar{a}_{0-} e^{i\left(\bar{\delta}_{0-}+\gamma\right)} = -\bar{A}_1 + \bar{A}_0$	$\bar{A} e^{i\left(\bar{\delta}_A+\gamma\right)}$
$\bar{B}^0 \to \pi^+ D^-_s$	$\bar{A}_{+s^-} = \bar{a} e^{i\left(\bar{\delta}+\gamma\right)}$	$\bar{T} e^{i\left(\bar{\delta}_T+\gamma\right)}$
$B^- \to \pi^0 D^-_s$	$\bar{A}_{0s^-} = (1/\sqrt{2}) \bar{A}_{+s^-}$	$(1/\sqrt{2}) \bar{T} e^{i\left(\bar{\delta}_T+\gamma\right)}$
$B^- \to \eta_8 D^-_s$	$\bar{A}_{8s^-} = \bar{a}_8 e^{i\left(\bar{\delta}_8+\gamma\right)}$	$\left((1/\sqrt{6}) T e^{i\bar{\delta}_T} - \sqrt{2/3} \bar{A} e^{i\bar{\delta}_A}\right) e^{i\gamma}$
$\bar{B}^0_s \to K^+ D^-_s$	$\bar{B}_{+s^-} = \bar{b}_s e^{i\left(\bar{\delta}_s+\gamma\right)}$	$\left(\bar{T} e^{i\bar{\delta}_T} + \bar{E} e^{i\bar{\delta}_E}\right) e^{i\gamma}$
$\bar{B}^0_s \to \pi^+ D^-$	$\bar{B}_{+-} = \bar{b}_{1/2} e^{i\left(\bar{\delta}_{1/2}+\gamma\right)} = \bar{B}_{1/2}$	$\bar{E} e^{i\left(\bar{\delta}_E+\gamma\right)}$
$\bar{B}^0_s \to \pi^0 \bar{D}^0$	$\bar{B}_{00} = (1/\sqrt{2}) \bar{b}_{1/2} e^{i\left(\bar{\delta}_{1/2}+\gamma\right)} =$ $= (1/\sqrt{2}) \bar{B}_{1/2}$	$(1/\sqrt{2}) \bar{E} e^{i\left(\bar{\delta}_E+\gamma\right)}$

Table 2: Amplitudes for $\Delta C = \pm 1$ and $\Delta S = -1$ decay modes of $\bar{B}$. $\Delta C = -1$, $\Delta S = -1$ decays.

correction to A_f by the procedure outlined above. Final state interactions considered from this point of view will be labelled as FSI (A). In an alternative point of view, labelled (B) one lumps all channels other the elastic channel in one category. In the random phase approximation of reference [11], one may take the parameter $\rho = \left[\overline{|A_n^2|}\right]^{1/2} / A_f$, $n \neq f$ nearly 1 for the color suppressed decays but ρ small for the two body decays dominated by tree graphs. In this paper the observational effects of final state interactions will be analyzed from the point of view (A).

2. Decay amplitudes decomposition

The amplitudes for various $\bar{B}$ decays are listed in table 1 and 2. They are characterized according to decay topologies: (1) a color-favored tree amplitude T, (2) a color-suppressed tree amplitude C, (3) an exchange amplitude E, and (4) an annihilation amplitude A. The isospin decomposition of relevant decays along with strong interaction phases are also given in table 1 and 2. Isospins and SU(3) symmetry gives the following relationships for various decay amplitudes

$$
\begin{aligned}
A_{-+} + A_{00} &= A_{-0} && (2.1)\\
B_{-s^+} - B_{-+} &= A_{-+} && (2.2)\\
\bar{A}_{-0} - \bar{A}_{0-} &= \bar{A}_{00} && (2.3)\\
\sqrt{2}\bar{A}_{0s} - \sqrt{6}\bar{A}_{8s^-} &= 2\bar{A}_{0-} && (2.4)\\
\bar{B}_{+s^-} - \bar{B}_{+-} &= \bar{A}_{+s^-} && (2.5)\\
\bar{B}_{+-} - \sqrt{6}\bar{B}_{80} &= 2\bar{A}_{00}\,. && (2.6)
\end{aligned}
$$

First we note that in the naive factorization ansatz, we have the following relations between the amplitudes of various topologies[14]

$$
\begin{aligned}
\frac{C}{T} &= \left(\frac{a_2}{a_1}\right)\frac{f_D F_0^{B-K}(m_D^2)(m_B^2-m_K^2)}{f_K F_0^{B-D}(m_K^2)(m_B^2-m_D^2)}\\
&\approx \left(\frac{a_2}{a_1}\right)(0\cdot 72) && (2.7)\\
\frac{E}{T} &= \left(\frac{a_2}{a_1}\right)\frac{f_{B_s} F_0^{D_s-K}(m_B^2)(m_{D_s}^2-m_K^2)}{f_K F_0^{B_s-D_s}(m_K^2)(m_{B_s}^2-m_{D_s}^2)}\\
&\approx \left(\frac{a_2}{a_1}\right)(0\cdot 08) && (2.8)\\
\frac{\bar{T}}{T} &\approx 0\cdot 72\sqrt{\rho^2+\eta^2} && (2.9)\\
\frac{\bar{C}}{\bar{T}} &= \left(\frac{a_2}{a_1}\right)\frac{f_D F_0^{B-K}(m_D^2)(m_B^2-m_K^2)}{f_{D_s} F_0^{B-\pi}(m_{D_s}^2)(m_B^2-m_\pi^2)}\\
&\approx \left(\frac{a_2}{a_1}\right) && (2.10)\\
\frac{\bar{A}}{\bar{T}} &= \frac{f_B F_0^{D-K}(m_B^2)(m_D^2-m_K^2)}{f_{D_s} F_0^{B-\pi}(m_{D_s}^2)(m_B^2-m_\pi^2)}\\
&\approx 0\cdot 08\,, && (2.11)
\end{aligned}
$$

where $a_2/a_1 \approx 0.2-0.3$. The numerical values have been obtained using $f_D \approx 200\,\text{MeV}$, $f_{D_s} \approx 240\,\text{MeV}$, $f_K \approx 158\,\text{MeV}$, $f_B \approx 180\,\text{MeV}$.

$$
\begin{aligned}
&\frac{F_0^{B-K}\left(m_D^2\right)}{F_0^{B-D}\left(m_K^2\right)} \approx 0.05 \approx \frac{F_0^{D_s-K}\left(m_B^2\right)}{F_0^{B_s-D_s}\left(m_K^2\right)}\\
&F_0^{B-K}\left(m_D^2\right) \approx F_0^{B-\pi}\left(m_{D_s}^2\right)\,. && (2.12)
\end{aligned}
$$

Since the amplitudes C $(\bar{C})$, E, $\bar{A}$ are suppressed relative to tree amplitude, they are subject to important corrections due to rescattering.

3. Rescattering

In order to calculate rescattering corrections and to obtain s-wave strong phases, we consider the scattering processes

$$\begin{aligned} P_a + \bar{D} &\to P_b + \bar{D} \\ P_a + D &\to P_b + D\,, \end{aligned}$$

where P_a is a pseudoscalar octet. Using SU(3), the scattering amplitude can be written as

$$M = \chi^\dagger \left[\bar{F}_1 \frac{1}{2}[\lambda_b, \lambda_a] + \bar{F}_2 \frac{1}{2}\{\lambda_b, \lambda_a\} + \bar{F}_3 \delta_{ba} \right] \chi\,, \tag{3.1}$$

where χ is an SU(3) triplet

$$\chi = \begin{pmatrix} \bar{D}^0 \\ D^- \\ D_s^- \end{pmatrix}. \tag{3.2}$$

For the process $P_a + D \to P_b + D$, we replace $\bar{F}_i$ $(i = 1, 2, 3)$ by F_i and χ by the triplet

$$\chi = \begin{pmatrix} D^0 \\ D^+ \\ D_s^+ \end{pmatrix}. \tag{3.3}$$

In order to express the scattering amplitudes in terms of Regge trajectories, it is convenient to define two amplitudes

$$\begin{aligned} M^+ &= P + f + A_2 \\ &= -C_P \frac{e^{-i\pi\alpha_P(t)/2}}{\sin \pi\alpha_P(t)/2} \left(\frac{s}{s_0}\right)^{\alpha_P(t)} + \\ &\quad + \left[-C_f \frac{1 + e^{-i\pi\alpha_f(t)}}{\sin \pi\alpha_f(t)} \left(\frac{s}{s_0}\right)^{\alpha_f(t)} - C_{A_2} \frac{1 + e^{-i\pi\alpha_{A_2}(t)}}{\sin \pi\alpha_{A_2}(t)} \left(\frac{s}{s_0}\right)^{\alpha_{A_2}(t)} \right] \end{aligned} \tag{3.4}$$

$$M^- = \rho + \omega = \left[C_\omega \frac{1 - e^{-i\pi\alpha_\omega(t)}}{\sin \pi\alpha_\omega(t)} + C_\rho \frac{1 - e^{-i\pi\alpha_\rho(t)}}{\sin \pi\alpha_\rho(t)} \right]. \tag{3.5}$$

Due to exchange degeneracy, for linear Regge trajectories

$$\alpha_\rho(t) = \alpha_{A_2}(t) = \alpha_\omega(t) = \alpha_f(t) = \alpha_0(t) + \acute{\alpha} t \tag{3.6}$$

$$C_f = C_\omega\,; \qquad C_{A_2} = C_\rho \tag{3.7}$$

$$C_\omega = C_\rho\,. \tag{3.8}$$

We take [13, 15] $\alpha_0 = 0.44 \approx 1/2$ and $\acute{\alpha} = 0.94\,\mathrm{GeV}^{-2} \approx 1\,\mathrm{GeV}$ and for the pomeron $\alpha_P(t) = \alpha_P(0) + \acute{\alpha}_P t$, $\alpha_P(0) = 1.08 \approx 1$ and $\acute{\alpha}_P \approx 0.25\,\mathrm{GeV}^{-2}$. In particular for the processes $K^-D^0 \to K^-D^0$ and $K^-\bar{D}^0 \to K^-\bar{D}^0$,we get

$$\begin{aligned}
M(K^-D^0 \to K^-D^0) &= P + (f-\omega) + (A_2 - \rho) \\
&= iC_P(s/s_0)e^{bt} - 2(C_\omega + C_\rho)\frac{1}{\sin\pi\alpha(t)}\left(\frac{s}{s_0}\right)^{\alpha(t)} \\
&= iC_P(s/s_0)e^{bt} - 4C_\rho\frac{1}{\sin\pi\alpha(t)}\left(\frac{s}{s_0}\right)^{\alpha(t)} \\
&= F_P + F_\rho \qquad (3.9) \\
M(K^-\bar{D}^0 \to K^-\bar{D}^0) &= P + (f+\omega) + (A_2 + \rho) \\
&= F_P + e^{-i\pi\alpha(t)}F_\rho = F_P + \bar{F}_\rho\,, \qquad (3.10)
\end{aligned}$$

where $b = \acute{\alpha}_P \ln(s/s_0)$. We take [11] $C_P = 5$. In order to estimate C_ρ, we note that SU(3) gives

$$\begin{aligned}
\gamma_{\rho K^+K^-} &= -\gamma_{\rho K^0\bar{K}^0} = \frac{1}{2}\gamma_{\rho\pi^+\pi^-} = \frac{1}{2}\gamma_0 \\
\gamma_{\omega K^+K^-} &= \gamma_{\omega K^0\bar{K}^0} = \frac{1}{2}\gamma_0 \\
\gamma_{\rho D^+D^-} &= -\gamma_{\rho D^0\bar{D}^0} = -\gamma_{\omega D^+D^-} = -\gamma_{\omega D^0\bar{D}^0}\,. \qquad (3.11)
\end{aligned}$$

For our purpose, we will take $\gamma_{\rho D^+D^-} \approx \gamma_{\rho K^+K^-} = \frac{1}{2}\gamma_0$, so that

$$C_\rho = \gamma_{\rho K^+K^-}\gamma_{\rho D^0\bar{D}^0} = -\frac{1}{4}\gamma_0^2\,. \qquad (3.12)$$

For γ_0^2, we use that value $\gamma_0^2 = 72$ as given in reference [15].

Now using eq. (3.1) and eqs. (3.9) and (3.10), we can express all the elastic or quasi-elastic scattering amplitudes in terms of the Regge amplitudes F_P and F_ρ. These amplitudes are given in table 3.

We are now in a position to discuss the rescattering corrections to the decay amplitudes. From eq. (1.3), the two particle unitarity gives [13, 15].

$$\begin{aligned}
DiscA(\bar{B}^0 \to \bar{K}^0D^0) &= \frac{1}{32\pi}\frac{|\vec{p}|}{s}\int d\Omega M^*(\bar{K}^0D^0 \to K^-D^+)A(\bar{B}^0 \to K^-D^+) \\
&\approx \frac{1}{16\pi s}\int_{-2|\vec{p}|^2}^{0} dtM^*(\bar{K}^0D^0 \to K^-D^+)A(\bar{B}^0 \to K^-D^+)\,, \qquad (3.13)
\end{aligned}$$

where we have put $|\vec{p}| \simeq \frac{1}{2}\sqrt{s}$. Now using table 3 and eqs. (3.9) and (3.12), we get

$$\begin{aligned}
DiscA(\bar{B}^0 \to \bar{K}^0D^0) &= \gamma_0^2\frac{1}{16\pi s}\frac{A(\bar{B}^0 \to K^-D^+)}{\sin\pi\alpha_0} \times (s/s_0)^{\alpha_0}\int_{-2|\vec{p}|^2}^{0} dte^{t\acute{\alpha}\ln s/s_0} \\
&= \frac{\gamma_0^2}{16\pi}(s/s_0)^{\alpha_0-1}\frac{1}{\ln(s/s_0)}A(\bar{B}^0 \to K^-D^+)\,, \qquad (3.14)
\end{aligned}$$

where in evaluating the integral in eq. (3.14), we have put $\sin\pi\alpha(t) = \sin\pi\alpha_0 = \sin\pi/2 = 1$ and $\acute{\alpha}s_0 = 1$ i.e. $s_0 = 1\,\mathrm{GeV}^2$.

Scattering Prpocesses	Scattering amplitude	Scattering amplitude in terms of Regge amplitudes F_P and F_ρ eqs. (3.9) and (3.10).
$K^-D^+ \to K^-D^+$	$-(2/3)F_2 + F_3$	$F_P : \bar{K}^0 D^0 \to \bar{K}^0 D^0$
$K^-D^0 \to K^-D^0$	$F_1 + (1/3)F_2 + F_3$	$F_P + F_\rho$
$K^-D^+ \rightleftharpoons \bar{K}^0 D^0$	$F_1 + F_2$	F_ρ
$K^-D_s^+ \to K^-D_s^+$	$-F_1 + (1/3)F_2 + F_3$	$F_P + e^{-i\pi\alpha(t)} F_\rho : \pi^- D^+ \to \pi^- D^+$
$K^-D_s^+ \to \pi^- D^+$	$-F_1 + F_2$	$e^{-i\pi\alpha(t)} F_\rho : \sqrt{2}\left(K^- D_s^+ \to \pi^0 D^0\right)$
$K^-\bar{D}^0 \to K^-\bar{D}^0$	$\bar{F}_1 + (1/3)\bar{F}_2 + \bar{F}_3$	$F_P + e^{-i\pi\alpha(t)} F_\rho : \bar{K}^0 D^- \to \bar{K}^0 D^-$
$\bar{K}^0 D^- \rightleftharpoons \bar{K}^0 D^-$	$\bar{F}_1 + \bar{F}_2$	$e^{-i\pi\alpha(t)} F_\rho$
$\bar{K}^0 \bar{D}^0 \to \bar{K}^0 \bar{D}^0$	$-(2/3)\bar{F}_2 + \bar{F}_3$	F_P
$\pi^+ D_s^- \to \pi^+ D_s^-$	$-(2/3)\bar{F}_2 + \bar{F}_3$	$F_P : \pi^0 D_s^- \to \pi^0 D_s^-$
$\eta_8 D_s^- \to \eta_8 D_s^-$	$(2/3)\bar{F}_2 + \bar{F}_3$	$F_P + \left(1 + e^{-i\pi\alpha(t)}\right) F_\rho$
$\pi^+ D_s^- \to \bar{K}^0 \bar{D}^0$	$-\bar{F}_1 + \bar{F}_2$	F_ρ
$\pi^0 D_s^- \to K^- \bar{D}^0$	$(1/\sqrt{2})\left(-\bar{F}_1 + \bar{F}_2\right)$	$(1/\sqrt{2}) F_\rho : -\left(\pi^0 D_s^- \to \bar{K}^0 D^-\right)$
$\eta_8 D_s^- \to \bar{K}^0 D^-$	$-(1/\sqrt{6})\left(3\bar{F}_1 + \bar{F}_2\right)$	$(1/\sqrt{6})\times$ $\times(1 - 2e^{-i\pi\alpha(t)}) F_\rho : \eta_8 D_s^- \to K^- \bar{D}^0$
$K^+ D_s^- \to K^+ D_s^-$	$\bar{F}_1 + (1/3)\bar{F}_2 + \bar{F}_3$	$F_P + e^{-i\pi\alpha(t)} F_\rho : \pi^+ D^- \to \pi^+ D^-$
$K^+ D_s^- \to \pi^+ D^-$	$\bar{F}_1 + \bar{F}_2$	$e^{-i\pi\alpha(t)} F_\rho : \sqrt{2}\left(K^+ D_s^- \to \pi^0 \bar{D}^0\right)$

Table 3: Scattering amplitudes for various scattering processes as given by SU(3) (eq. (3.1)). The last column gives the amplitudes in terms of Regge exchanges $F_P = iC_P(s/s_0)e^{bt}$, $F_\rho = \gamma_0^2(1/\sin\pi\alpha(t))(s/s_0)^{\alpha(t)}$, $C_P \approx 5$, $\gamma_0^2 \approx 72$.

We now use dispersion relation [13, 15, 16] to obtain

$$A(\bar{B}^0 \to \bar{K}^0 D^0)_{FSI} = \frac{\gamma_0^2}{16\pi} \frac{A(\bar{B}^0 \to K^- D^+)}{\ln(m_B^2/s_0)} \frac{\sqrt{s_0}}{m_B} \frac{1}{\pi} \int_{(m_D+m_K)^2}^{\infty} (s/s_0)^{\alpha_0 - 1} \frac{ds}{s - m_B^2}, \tag{3.15}$$

where in $\ln(s/s_0)$, we have put $s = m_B^2$. Noting that $\alpha_0 \approx 1/2$, we get

$$\begin{aligned} A(\bar{B}^0 \to \bar{K}^0 D^0)_{FSI} &= \frac{\gamma_0^2}{16\pi} \frac{1}{\ln(m_B^2/s_0)} \frac{\sqrt{s_0}}{m_B} \frac{1}{\pi} \left[i\pi + \ln \frac{1+x}{1-x} \right] A(\bar{B}^0 \to K^- D^+) \\ &\equiv \epsilon e^{i\theta} A(\bar{B}^0 \to K^- D^+), \end{aligned} \tag{3.16}$$

where

$$\begin{aligned} x &= \frac{m_D + m_K}{m_B} \simeq 0 \cdot 447, m_B = 5 \cdot 279 \\ \epsilon &= \frac{\gamma_0^2}{16\pi} \frac{1}{\ln(m_B^2/s_0)} \frac{\sqrt{s_0}}{m_B} \left[1 + \frac{1}{\pi^2} \left(\ln \frac{1+x}{1-x} \right)^2 \right]^{1/2} \\ &= \gamma_0^2 \left[1 \cdot 18 \times 10^{-3} \right] \simeq 0 \cdot 08 \\ \theta &= \tan^{-1} \left[\frac{\pi}{\ln \frac{1+x}{1-x}} \right] \simeq 73^0 . \end{aligned} \tag{3.17}$$

From eqs. (2.2), (2.6) and table 3, following the same procedure, we can eaisly calculate the rescattering corrections for other decays. Hence after taking into account rescattering corrections to the decay amplitudes, we get

$$\begin{aligned}
A_{00} &= a_{00}e^{i\delta_{00}} + \epsilon e^{i\theta} a_{-+} e^{i\delta_{-+}} && (3.18)\\
A_{-+} &= a_{-+}e^{i\delta_{-+}} && (3.19)\\
A_{-0} &= a_{-0}e^{i\delta_{-0}} + \epsilon e^{i\theta} a_{-+} e^{i\delta_{-+}} && (3.20)\\
B_{-s^+} &= b_s e^{i\delta_s} && (3.21)\\
B_{-+} &= b_{1/2}e^{i\delta_{1/2}} + \frac{1}{2}(1+i)\epsilon e^{i\theta} b_s e^{i\delta_s} && (3.22)\\
\bar{A}_{00} &= (\bar{a}_{00}e^{i\bar{\delta}_{00}} + \epsilon e^{i\theta}\bar{a}e^{i\bar{\delta}})e^{i\gamma} && (3.23)\\
\bar{A}_{-0} &= \left(\bar{a}_{-0}e^{i\bar{\delta}_{-0}} + \frac{1}{2}\epsilon\left(1-\frac{i}{3}\right)\bar{a}e^{i\bar{\delta}}\right)e^{i\gamma} && (3.24)\\
\bar{A}_{0-} &= \left(\bar{a}_{0-}e^{i\bar{\delta}_{0-}} - \frac{1}{2}\epsilon\left(1+\frac{i}{3}\right)\bar{a}e^{i\bar{\delta}}\right)e^{i\gamma} && (3.25)\\
B_{+s^-} &= \bar{b}_s e^{i\bar{\delta}_s} && (3.26)\\
\bar{B}_{-+} &= \bar{b}_{1/2}e^{i\bar{\delta}_{1/2}} + \frac{1}{2}(1+i)\epsilon e^{i\theta}\bar{b}_s e^{i\bar{\delta}_s}\,. && (3.27)
\end{aligned}$$

The phase factor i arises due to the phase factor $e^{i\pi\alpha(t)}$ ($\bar{F}_\rho = \ e^{i\pi\alpha(t)}F$) [we can also write $(1+i)(1/2) = (1/\sqrt{2})e^{i\pi/4}$, $(1\pm i/3) = (\sqrt{10}/3)e^{\pm i\phi}$, $\phi = \tan^{-1} 1/3 = 18^0$].

4. Strong interaction phase shifts

For the s-wave scattering, the $l = 0$ partial wave scattering amplitude f is given by

$$f = \frac{1}{16\pi s}\int_{-4\vec{p}^2}^{0} M(s,t)dt\,. \tag{4.1}$$

Hence from eqs. (3.9) and (3.10), we get for $\sqrt{s} = 5\cdot 279\,\mathrm{GeV}$,

$$\begin{aligned}
f_P &= \frac{1}{16\pi s}\frac{iC_P}{b}\,(s/s_0) \simeq 0.12i && (4.2)\\
f_\rho &= \frac{\gamma_0^2}{16\pi}\frac{1}{\ln(s/s_0)}\,(s/s_0)^{1/2} \simeq 0.08 && (4.3)\\
\bar{f}_\rho &= \frac{\gamma_0^2}{16\pi}(-i)\frac{1}{\ln(s/s_0) - i\pi}\,(s/s_0)^{1/2} \simeq 0.04 + 0.04i\,. && (4.4)
\end{aligned}$$

Using table 3 and eqs. (4.2)–(4.4), we can determine the s-wave scattering amplitude f and S-matrix, $S = 1 + 2if \equiv \eta e^{2i\Delta}$ for each individual scattering process. They are given in table 4.

Now, using eq. (1.5) we get

$$\mathrm{Re}\,A_f\left(1 - \eta e^{-2i\Delta}\right) - i\,\mathrm{Im}\,A_f\left(1 + \eta e^{2i\Delta}\right) = \frac{1}{i}\sum_{n\neq f} A_n S^*_{nf}\,. \tag{4.5}$$

Scattering process	f	S	η	Δ	$\frac{1-\eta}{1+\eta}$
$K^-D^0 \to K^-D^0$	$0.08 + 0.12i$	$0.76 + 0.16i$	0.78	6^0	0.12
$K^-D^+ \to K^-D^+$	$0.12i$	0.76	0.76	0	0.14
$\bar{K}^0D^0 \to \bar{K}^0D^0$	$0.12i$	0.76	0.76	0	0.14
$K^-D_s^+ \to K^-D_s^+$	$0.04 + 0.16i$	$0.68 + 0.08i$	0.68	3.5^0	0.19
$\pi^-D^+ \to \pi^-D^+$	$0.04 + 0.16i$	$0.68 + 0.08i$	0.68	3.5^0	0.19
$\bar{K}^0\bar{D}^0 \to \bar{K}^0\bar{D}^0$	$0.12i$	0.76	0.76	0	0.14
$\pi^+D_s^- \to \pi^+D_s^-$	$0.12i$	0.76	0.76	0	0.14
$\pi^0D_s^- \to \pi^0D_s^-$	$0.12i$	0.76	0.76	0	0.14
$K^-\bar{D}^0 \to K^-\bar{D}^0$	$0.04 + 0.16i$	$0.68 + 0.08i$	0.68	3.5^0	0.19
$\bar{K}^0D^- \to \bar{K}^0D^-$	$0.04 + 0.16i$	$0.68 + 0.08i$	0.68	3.5^0	0.19
$K^+D_s^- \to K^+D_s^-$, $\pi^+D^- \to \pi^+D^-$	$0.04 + 0.16i$	$0.68 + 0.08i$	0.68	3.5^0	0.19

Table 4: Partial wave $l = 0$ scattering amplitude f for elastic scattering; $S = 1 + 2if = \eta e^{2i\Delta}$.

Taking the absolute square of eq. (4.5), we obtain, writing $A_f = |A_f|\, e^{i\delta_f}$:

$$|A_f|^2 \left[\left(1+\eta^2\right) - 2\eta\cos 2\left(\delta_f - \theta\right)\right] = \sum_{n,\acute{n}\neq f} A_n S^*_{nf} A^*_{\acute{n}} S_{\acute{n}f}\,. \tag{4.6}$$

Using the random phase approximation of reference [11]:

$$\sum_{n,\acute{n}\neq f} A_n S^*_{nf} A^*_{\acute{n}} S_{\acute{n}f} = \sum_{n\neq f} |A_n|^2 |S_{nf}|^2 = \overline{|A_n|}^2 \left(1-\eta^2\right) \tag{4.7}$$

we obtain from eqs. (4.6) and (4.7):

$$\tan^2\left(\delta_f - \Delta\right) = \left(\frac{1-\eta}{1+\eta}\right)\frac{\left[\rho^2 - \left((1-\eta)/(1+\eta)\right)\right]}{\left[1-\rho^2\left((1-\eta)/(1+\eta)\right)\right]} \tag{4.8}$$

$$\rho^2 = \frac{\overline{|A_n|}^2}{|A_f|^2}\,, \qquad \frac{1-\eta}{1+\eta} \le \rho^2 \le \frac{1+\eta}{1-\eta}\,. \tag{4.9}$$

Except for the parameter ρ, everything is known from the table 4. For the moment let us evaluate the final state phase shifts δ_f from eq. (4.9) for three values of ρ^2 viz $\rho^2 = (1-\eta)\,/\,(1+\eta)$, $0 \cdot 25$ and 1. These phase shifts are tabulated in table 5 for various decays.

The following remarks are in order. Irrespective of parameter ρ, it follows from the above analysis that

$$\begin{aligned} \bar{\delta}_s &= \delta_s \\ \bar{\delta}_{1/2} &= \delta_{1/2}\,. \end{aligned} \tag{4.10}$$

It is reasonable to assume that ρ is of the same order for all the favored decays; it may be different for the suppressed decays but within a set it does not differ much. Under this assumption it follows from table 5

$$\begin{aligned} \bar{\delta} &= \delta_{-+} \\ \bar{\delta}_{-0} &= \bar{\delta}_{0-} \\ \bar{\delta}_{00} &= \delta_{00}\,. \end{aligned} \tag{4.11}$$

Further if we take the point of view consistent with that of reference [8] i.e. for the decay amplitudes dominated by tree graphs the final state interactions are negligible, (ρ: small) then we may conclude

$$\begin{aligned} \delta_{-+} &= \bar{\delta} \leq 7^0 \\ \delta_{-0} &\leq 13^0 \\ \delta_s &= \bar{\delta}_s \leq 10^0\,. \end{aligned} \tag{4.12}$$

To proceed further, we first consider the case (A): For the color suppressed decays, since we have already taken into account the rescattering corrections c.f. eqs. (3.22) and (3.27), it is reasonable to take the same value of ρ for all the decays ($\rho \ll 1$). Then from table 5, we have

$$\begin{aligned} \bar{\delta}_{00}=\delta_{00} &= \delta_{-+} \leq 7^0 \\ \bar{\delta}_{-0} &= \bar{\delta}_{0-} \leq 10^0 \\ \bar{\delta}_{1/2} &= \delta_{1/2} \leq 10^0 \end{aligned} \tag{4.13}$$

For the case (B): The phase shifts for color favored decays will be small as given in eq. (4.12); but for the color suppressed decays ρ would be of the order 1 and the phase shifts for these decays will be in the range of 20^0 as given in table 5. For this case, in eqs. (3.22) and (3.27), ϵ will be taken to zero as the effect of rescattering is supposed to be reflected in the parameter $\rho = 1$.

5. Observational consequences of final state interactions (FSI)

We note that the decay amplitudes after taking into account FSI are given in eqs. (3.22) and (3.27). From these equations, we obtain (neglecting terms of order ϵ^2)

$$a^2_{-0} = a^2_{-+} + a^2_{00} + 2a_{-+}a_{00}\cos\left(\delta_{-+} - \delta_{00}\right) \tag{5.1}$$

$$\Gamma\left(B^- \to K^- D^0\right) = a^2_{-0} + 2\epsilon a_{-0}a_{-+}\cos\left(\theta + \delta_{-+} - \delta_{-0}\right) \tag{5.2}$$

$$\Gamma\left(\bar{B}^0 \to \bar{K}^0 D^0\right) = a^2_{00} + 2\epsilon a_{00}a_{-+}\cos\left(\theta + \delta_{-+} - \delta_{00}\right) \tag{5.3}$$

$$\Gamma\left(\bar{B}^0 \to K^- D^+\right) = a^2_{-+} \tag{5.4}$$

$$\bar{a}^2_{-0} = \bar{a}^2_{00} + \bar{a}^2_{0-} + 2\bar{a}_{00}\bar{a}_{0-}\cos\left(\bar{\delta}_{00} - \bar{\delta}_{0-}\right) \tag{5.5}$$

$$\Gamma\left(B^- \to K^- \bar{D}^0\right) \simeq \bar{a}^2_{-0} + \frac{\sqrt{10}}{3}\epsilon\bar{a}_{-0}\bar{a}\cos\left(\theta - \phi + \bar{\delta} - \bar{\delta}_{-0}\right) \tag{5.6}$$

$$\Gamma\left(B^- \to \bar{K}^0 D^-\right) \simeq \bar{a}^2_{0-} - \frac{\sqrt{10}}{3}\epsilon\bar{a}_{0-}\bar{a}\cos\left(\theta + \phi + \bar{\delta} - \bar{\delta}_{0-}\right) \tag{5.7}$$

$$\Gamma\left(\bar{B}^0 \to \bar{K}^0 \bar{D}^0\right) \simeq \bar{a}^2_{00} + 2\epsilon\bar{a}_{00}\bar{a}\cos\left(\theta + \bar{\delta} - \bar{\delta}_{00}\right) \tag{5.8}$$

$$\Gamma\left(B^- \to \pi^0 D_s^-\right) = \bar{a}^2\,. \tag{5.9}$$

phase shift in degree ρ	$\sqrt{\frac{1-\eta}{1+\eta}}$	0.5	1
δ_{-0}	6	13	25
δ_{-+}	0	7	20
δ_{00}	0	7	20
δ_s	3	10	23
$\delta_{1/2}$	3	10	23
$\bar{\delta}_{00}$	0	7	20
$\bar{\delta}$	0	7	20
$\bar{\delta}_{-0}$	3	10	23
$\bar{\delta}_{0-}$	3	10	23
$\bar{\delta}_s$	3	10	23
$\bar{\delta}_{1/2}$	3	10	23

Table 5: Final state strong interaction phase shift δ_f for $\rho = \sqrt{(1-\eta)/(1+\eta)}$, 0.5 and 1.

First we note from eqs. (3.22), (3.27), (5.1) (5.5) that in the absence of rescattering the amplitudes $|A_{-0}|$, $|A_{-+}|$, $|A_{00}|$, and $\left|\bar{A}_{-0}\right|$, $\left|\bar{A}_{00}\right|$, $\left|\bar{A}_{0-}\right|$ will form two closed triangles. Any deviation from triangular relations would indicate rescattering.

Now using the factorization(see table 1 and 2) and eqs. (2.7)–(2.11), we obtain $a_{00}/a_{-+} = C/T \approx 0.72\,(a_2/a_1)$, $\bar{a}_{00}/\bar{a} = \bar{C}/\bar{T} \approx (a_2/a_1)$, $\bar{a}_{0-}/\bar{a} = \bar{A}/\bar{T} \approx 0.08$ and noting that $\epsilon = 0.08$, $a_2/a_1 \sim \lambda = 0 \cdot 22$, $\theta = 73^0$ and $\phi = 18^0$, we note that FSI corrections are in the range of $15 - 20\%$, except for $B^- \to \bar{K}^0 D^-$, (c.f. eq. (5.3) where it is almost zero, since $\theta + \phi \simeq 90^0$. However we note that the effect of FSI is of considerable importance for the decays $\bar{B}_s^0 \to \pi^- D^+$ and $\bar{B}_s^0 \to \pi^+ D^-$ as both these decays in the absence of FSI are extremely suppressed as these decays occur through W-exchange diagram (c.f. table 1 and 2). Taking into account FSI, we obtain from eqs. (3.22), (3.27) and (2.8)

$$\begin{aligned}
\frac{\Gamma\left(\bar{B}_s^0 \to \pi^- D^+\right)}{\Gamma\left(\bar{B}_s^0 \to K^- D_s^+\right)} &= \frac{b_{1/2}^2}{b_s^2}\left[1 + \sqrt{2}\epsilon\frac{b_s}{b_{1/2}}\cos\left(\theta + \frac{\pi}{4} + \delta_s - \delta_{1/2}\right) + \frac{1}{2}\epsilon^2\frac{b_s^2}{b_{1/2}^2}\right] \\
&\simeq \sqrt{2}\epsilon\,(E/T)\cos\left(\theta + \frac{\pi}{4} + \delta_s - \delta_{1/2}\right) + \frac{1}{2}\epsilon^2 \\
&\simeq 2.3 \times 10^{-3} = \frac{\Gamma\left(\bar{B}_s^0 \to \pi^+ D^-\right)}{\Gamma\left(\bar{B}_s^0 \to K^+ D_s^-\right)}\,. \qquad (5.10)
\end{aligned}$$

In the absence of FSI, this ratio has the value $3 \cdot 1 \times 10^{-4}$.

We now discuss the effect of FSI on CP-asymmetry. We define

$$\begin{aligned}
\mathcal{A}_{\mp} &= \frac{\Gamma\left(B^- \to K^- D_{\mp}^0\right) - \Gamma\left(B^+ \to K^+ D_{\mp}^0\right)}{\Gamma\left(\bar{B}^0 \to K^- D^+\right)} \\
&= \pm 2\sin\gamma\frac{\left[\mathrm{Re}\,A_{-0}\,\mathrm{Im}\,\bar{A}_{-0} - \mathrm{Im}\,A_{-0}\,\mathrm{Re}\,\bar{A}_{-0}\right]}{\left|A_{-+}\right|^2} \qquad (5.11)
\end{aligned}$$

$$\mathcal{R}_{\mp} = \frac{\Gamma\left(B^- \to K^- D^0_{\mp}\right) + \Gamma\left(B^+ \to K^+ D^0_{\mp}\right)}{\Gamma\left(\bar{B}^0 \to K^- D^+\right)}$$
$$= \frac{\left\{\left|A_{-0}\right|^2 + \left|\bar{A}_{-0}\right|^2 \mp 2\cos\gamma\left[\mathrm{Re}\, A_{-0}\, \mathrm{Re}\, \bar{A}_{-0} + \mathrm{Im}\, A_{-0}\, \mathrm{Im}\, \bar{A}_{-0}\right]\right\}}{\left|A_{-+}\right|^2}, \qquad (5.12)$$

where $D^0_{\mp} = \frac{(D^0 \mp \bar{D}^0)}{\sqrt{2}}$. For $\bar{B}^0$ and B^0 decays, $\mathcal{A}^0_{\mp}$, $\mathcal{R}^0_{\mp}$ can be obtained by changing $A_{-0} \to A_{00}$ and $\bar{A}_{-0} \to \bar{A}_{00}$ in eqs. (5.11), (5.12). Then using eqs. (3.22) and (3.27), we get

$$\mathcal{A}_{\mp} = \pm 2\sin\gamma\left[-f\bar{r}\sin(\delta_{-0} - \bar{\delta}_{-0}) - \epsilon\bar{r}\sin(\theta + \delta_{-+} - \bar{\delta}_{-0}) + \right.$$
$$\left. + \frac{\sqrt{10}}{6}\epsilon f \bar{f}\sin(\theta - \phi + \bar{\delta} - \delta_{-0})\right] \qquad (5.13)$$

$$\mathcal{R}_{-} - \mathcal{R}_{+} = \mp 4\cos\gamma\left[f\bar{r}\cos(\delta_{-0} - \bar{\delta}_{-0}) + \epsilon\bar{r}\cos(\theta + \delta_{-+} - \bar{\delta}_{-0}) + \right.$$
$$\left. + \frac{\sqrt{10}}{6}\epsilon f \bar{f}\cos(\theta - \phi + \bar{\delta} - \delta_{-0})\right] \qquad (5.14)$$

$$\mathcal{A}^0_{\mp} = \pm 2\sin\gamma\left[-r_0\bar{r}_0\sin(\delta_{00} - \bar{\delta}_{00}) + \epsilon\bar{f}r_0\sin(\theta + \bar{\delta} - \delta_{00}) - \right. \qquad (5.15)$$
$$\left. - \epsilon\bar{r}_0\sin(\theta + \delta_{-+} - \bar{\delta}_{00}) + \epsilon^2\bar{f}\sin(\bar{\delta} - \delta_{-+})\right]$$

$$\mathcal{R}^0_{-} - \mathcal{R}^0_{+} = \mp 4\cos\gamma\left[r_0\bar{r}_0\cos(\delta_{00} - \bar{\delta}_{00}) + \epsilon\bar{f}r_0\cos(\theta + \bar{\delta} - \delta_{00}) + \right.$$
$$\left. + \epsilon\bar{r}_0\cos(\theta + \delta_{-+} - \bar{\delta}_{00})\right]$$
$$\simeq \mp 4\cos\gamma\left[r_0\bar{r}_0\cos(\delta_{00} - \bar{\delta}_{00})\right], \qquad (5.16)$$

where

$$f = \frac{a_{-0}}{a_{-+}} = (1 + C/T) \simeq 1.22$$
$$\bar{r} = \frac{\bar{a}_{-0}}{a_{-+}} = \left(\frac{\bar{C} + \bar{A}}{\bar{T}}\right)\frac{\bar{T}}{T} = \sqrt{\rho^2 + \eta^2}\,(0.72) \times 0.30$$
$$r_0 = \frac{a_{00}}{a_{-+}} = \frac{C}{T} = \left(\frac{a_2}{a_1}\right)$$
$$\bar{r}_0 = \frac{\bar{a}_{00}}{a_{-+}} = \frac{\bar{C}}{\bar{T}}\left(\frac{\bar{T}}{T}\right) = \sqrt{\rho^2 + \eta^2}\left(\frac{a_2}{a_1}\right)(0.72)$$
$$\bar{f} = \frac{\bar{a}}{a_{-+}} = \frac{\bar{T}}{T} = \sqrt{\rho^2 + \eta^2}\,(0.72)\,. \qquad (5.17)$$

As is clear from eqs. (5.13) and (5.15), the FSI corrections tend to cancel each other; in $\mathcal{A}^0_{\mp}$ the cancellation is almost complete and one gets $\mathcal{A}^0_{\mp} \simeq 0$, where as for $\mathcal{A}\sin\gamma$. From eqs. (5.14) and (5.16), using eq. (5.17) and phase shifts from table 5 (cf first column), we get

$$\begin{aligned}\mathcal{R}_- - \mathcal{R}_+ &\simeq (0.42)\cos\gamma \\ \mathcal{R}_-^0 - \mathcal{R}_+^0 &\simeq (0.23)\cos\gamma\,. \end{aligned} \tag{5.18}$$

Let us now discuss the direct CP-violation for B_s-decay. Defining $B_\mp^0 = \frac{1}{\sqrt{2}}\left(B_s^0 \mp \bar{B}_s^0\right)$, we get using eqs. (3.21) and (3.27)

$$\frac{2\left|A\left(B_\mp \to K^+D_s^-\right)\right|^2 - b_s^2 - \bar{b}_s^2}{2b_s\bar{b}_s} = \mp\left[\cos\left(\gamma - \delta_s + \bar{\delta}_s\right)\right] \tag{5.19}$$

$$\frac{2\left|A\left(B_\mp \to K^-D_s^+\right)\right|^2 - b_s^2 - \bar{b}_s^2}{2b_s\bar{b}_s} = \mp\left[\cos\left(\gamma + \delta_s - \bar{\delta}_s\right)\right]. \tag{5.20}$$

Since $\delta_s = \bar{\delta}_s$, it implies

$$\Gamma\left(B_\mp \to K^+D_s^-\right) = \Gamma\left(B_\mp \to K^-D_s^+\right). \tag{5.21}$$

Our result $\delta_s = \bar{\delta}_s$, has important implication for determining the phase γ discussed in reference [17].

Finally we discuss the time dependent analysis of B decays to get information about weak phase γ. Following the well known procedure [18], [19] the time dependent decay rate for $B^0(t)$ and $\bar{B}^0(t)$ are given by

$$\begin{aligned}\mathcal{A}(t) &\equiv \frac{[\Gamma_f(t) + \Gamma_{\bar{f}}(t)] - [\bar{\Gamma}_f(t) + \bar{\Gamma}_{\bar{f}}(t)]}{\Gamma_f(t) + \bar{\Gamma}_f(t)} \\ &\simeq -2\frac{\sin(\Delta m_B t)\sin(2\beta+\gamma) \times [\langle f|H|B^0\rangle^* \langle f|H|\bar{B}^0\rangle + \langle f|H|\bar{B}^0\rangle^* \langle f|H|B^0\rangle]}{|\langle f|H|B^0\rangle|^2 + |\langle f|H|\bar{B}^0\rangle|^2}\end{aligned} \tag{5.22}$$

and

$$\begin{aligned}\mathcal{F}(t) &= \frac{[\Gamma_f(t) + \Gamma_{\bar{f}}(t)] - [\bar{\Gamma}_f(t) + \bar{\Gamma}_{\bar{f}}(t)]}{\Gamma_f(t) + \bar{\Gamma}_f(t)} \\ &= -2\frac{\left[\begin{array}{c}\cos(\Delta m_B t)[|\langle f|H|\bar{B}^0\rangle|^2 - |\langle f|H|B^0\rangle|^2] + \sin(\Delta m_B t)\cos(2\beta+\gamma) \\ [\langle f|H|B^0\rangle^* \langle f|H|\bar{B}^0\rangle - \langle f|H|\bar{B}^0\rangle^* \langle f|H|B^0\rangle]\end{array}\right]}{|\langle f|H|B^0\rangle|^2 + |\langle f|H|\bar{B}^0\rangle|^2},\end{aligned} \tag{5.23}$$

where

$$\Gamma_{f,\bar{f}}(t) \equiv \Gamma\left(B^0(t) \to f, \bar{f}\right), \bar{\Gamma}_{f,\bar{f}}(t) \equiv \Gamma\left(\bar{B}^0(t) \to f, \bar{f}\right). \tag{5.24}$$

Taking $f \equiv K_s D^0$ and $\bar{f} \equiv K_s \bar{D}^0$ and using eqs. (3.27) and (4.1) we get

$$\begin{aligned}\mathcal{A}(t) = -4&\frac{\Gamma\left(\bar{B}^0 \to K^-D^+\right)(a_2/a_1)(C/T)\sqrt{\rho^2+\eta^2}}{\Gamma\left(\bar{B}^0 \to \bar{K}^0D^0\right) + \Gamma\left(\bar{B}^0 \to \bar{K}^0\bar{D}^0\right)} \times \\ &\times [\sin(\Delta m_B t)\sin(2\beta+\gamma) \times Y]\end{aligned} \tag{5.25}$$

$$\begin{aligned}\mathcal{F}(t) + 2\cos(\Delta m_B t)&\frac{\Gamma\left(\bar{B}^0 \to \bar{K}^0D^0\right) - \Gamma\left(\bar{B}^0 \to \bar{K}^0\bar{D}^0\right)}{\Gamma\left(\bar{B}^0 \to \bar{K}^0D^0\right) + \Gamma\left(\bar{B}^0 \to \bar{K}^0\bar{D}^0\right)} = \\ &= -4\left[\begin{array}{c}\frac{\Gamma\left(\bar{B}^0\to K^-D^+\right)(a_2/a_1)(C/T)\sqrt{\rho^2+\eta^2}}{\Gamma\left(\bar{B}^0\to\bar{K}^0D^0\right)+\Gamma\left(\bar{B}^0\to\bar{K}^0\bar{D}^0\right)} \\ \sin(\Delta m_B t)\cos(2\beta+\gamma) \times Z\end{array}\right],\end{aligned} \tag{5.26}$$

where

$$Y = \Big[\cos(\delta_{00} - \bar{\delta}_{00}) + \epsilon(a_1/a_2)\cos(\theta + \bar{\delta} - \delta_{00}) + \epsilon(a_1/a_2)\epsilon\cos(\theta + \delta_{-+} - \bar{\delta}_{00}) + \\ + \epsilon^2(a_1/a_2)^2\cos(\bar{\delta} - \delta_{-+})\Big] \tag{5.27}$$

$$Z = \left[\begin{array}{c} \sin(\delta_{00} - \bar{\delta}_{00}) + \epsilon(a_1/a_2)\sin(\theta + \bar{\delta} - \delta_{00}) + \\ \epsilon(a_1/a_2)\sin(\theta + \delta_{-+} - \bar{\delta}_{00}) + \epsilon^2(a_1/a_2)^2\sin(\bar{\delta} - \delta_{-+}) \end{array}\right] \tag{5.28}$$

From eqs. (5.25) and (5.26) it follows that the experimental measurements of $\mathcal{A}(t)$ and $\mathcal{F}(t)$ would give $\sin(2\beta+\gamma)Y$ and $\cos(2\beta+\gamma)Z$. Note that eqs. (5.25) and (5.26) directly gives $\tan(2\beta+\gamma)Y/Z$. However, the relations $\bar{\delta} = \delta_{-+}$, $\delta_{00} = \bar{\delta}_{00}$ do not depend on the detail of the model. In this case

$$Z = 2\epsilon(a_1/a_2)\sin\theta \approx 0.69 \tag{5.29}$$

$$Y = 1 + 2\epsilon(a_1/a_2)\cos\theta + \epsilon^2(a_1/a_2)^2 \approx 1.34\,, \tag{5.30}$$

where, we have used $\epsilon = 0.08$, $\theta = 73^0$ and $a_2/a_1 \approx 0.22$ to give an order of magnitude for Y and Z. It is clear that the dominant contribution to Z comes from the rescattering, where for Y, the rescattering corrections are in the range of 33%.

For time dependent decays of B_s^0, one can get

$$\mathcal{A}_s(t) \equiv \frac{\Gamma_{f_s}(t) - \Gamma_{\bar{f}_s}(t)}{\Gamma_{\bar{f}_s}(t) + \bar{\Gamma}_{f_s}(t)} = \frac{b_s\bar{b}_s}{b_s^2 + \bar{b}_s^2}\sin(\Delta m_{B_s}t)[S + \bar{S}] \tag{5.31}$$

$$\begin{aligned} \mathcal{F}_s(t) &\equiv \frac{[\Gamma_{f_s}(t) + \bar{\Gamma}_{\bar{f}_s}(t)] - [\Gamma_{\bar{f}_s}(t) + \bar{\Gamma}_{f_s}(t)]}{\Gamma_{\bar{f}_s}(t) + \bar{\Gamma}_{f_s}(t)} \\ &= 2[(b_s^2 - \bar{b}_s^2)\cos(\Delta m_{B_s}t) + b_s\bar{b}_s(S - \bar{S})\sin(\Delta m_{B_s}t)]\,, \end{aligned} \tag{5.32}$$

where $f_s = K^+D_s^-$, $\bar{f}_s = K^-D_s^+$ and

$$\begin{aligned} S &= \sin(2\phi_{Ms} + \gamma + \delta_s - \bar{\delta}_s) \\ \bar{S} &= \sin(2\phi_{Ms} + \gamma - \delta_s + \bar{\delta}_s)\,. \end{aligned} \tag{5.33}$$

Since b_s^2 and $\bar{b}_s^2$ are given by the decay widths $\Gamma\left(\bar{B}_s^0 \to K^-D_s^+\right)$ and $\Gamma\left(\bar{B}_s^0 \to K^+D_s^-\right)$ respectively, it is clear from eqs. (5.31) and (5.32) that it is possible to determine S and $\bar{S}$ from the experimental value of $\mathcal{A}_s(t)$ and $\mathcal{F}_s(t)$. However for B_s^0, $\phi_{Ms} \simeq 0$. But $\delta_s = \bar{\delta}_s$ follows from general arguments (cf. eq. (4.10)), it is therefore reasonable to use $\delta_s = \bar{\delta}_s$. In this case $S = \bar{S}$,it is then possible to determine $\sin\gamma$, using eq. (5.31).

Finally we give an estimate of the CP asymmetry parameter $\mathcal{A}(t)$ and $\mathcal{A}_s(t)$, using $\sqrt{\rho^2+\eta^2} = 0.36$. Then we find from eqs. (5.27), (5.29) and (5.31)

$$\begin{aligned} \mathcal{A}(t) &\approx -4\frac{\sqrt{\rho^2+\eta^2}}{1+\rho^2+\eta^2}[\sin(\Delta m_B t)\sin(2\beta+\gamma)\times 1.34] \\ &\approx -1.94\sin(\Delta m_B t)\sin(2\beta+\gamma) \\ \mathcal{A}_s(t) &\approx 2\frac{\sqrt{\rho^2+\eta^2}\bar{T}/T}{1+(\rho^2+\eta^2)\,\bar{T}/T}\sin(\Delta m_{B_s}t)\sin\gamma \\ &\approx 0.49\sin(\Delta m_{B_s}t)\sin\gamma\,. \end{aligned}$$

To conclude: The rescattering corrections are of the order of $15 - 20\%$; except for $\bar{B}_s^0 \to \pi^- D^+$ and $\bar{B}_s^0 \to \pi^+ D^-$ where they are greater than the decay amplitude given by W-exchange graph. The direct CP-asymmetry parameter is of the order of $10^{-3} \sin\gamma$ for $B^- \to K^- D^0_\mp$ and $B^+ \to K^+ D^0_\mp$ decays as the rescattering correction tend to cancel each other for these decays. But for the time dependent CP-asymmetry we get the value $-1.94 \sin(2\beta + \gamma)$ for $B^0 \to K_s D^0$, $K_s \bar{D}^0$ decays. For $B_s^0 \to K^\pm D_s^\mp$ decays our analysis gives the strong phase shifts $\delta_s = \bar{\delta}_s$ and we get time dependent CP-asymmetry of the order $(0 \cdot 49) \sin\gamma$ which may be used to extract the weak phase γ in future experiments. Finally the formalism developed for final state interactions in this paper is also applicable for the $\Delta S = 0$, $\Delta C = \pm 1$ B-decays

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